6th Week Wednesday

29 Mart 2023 Çarşamba 14:24

Make-up lesson → Sunday 11:30 - 12:30

$A_{n\times n}$ → coef. matrix $Ax = b$ or $Ax = 0$

* If $RREF(A) = I_n \iff A^{-1}$ exists $\iff$ $Ax=b$ has a unique soln. $= A^{-1}b$ $\iff \det(A) \neq 0$
$Ax = 0$ " " " " $=$ the trivial solution $\Rightarrow 0$

* If $RREF(A) \neq I_n \iff A^{-1}$ does not exist $\iff$ $Ax=b$ has either inf. many solutions or No solutions $\iff \det(A) = 0$
$Ax = 0$ has infinitely many solutions.

matrices → SLE
det

Cramer's Rule

$Ax = b$, $A_{n\times n}$ $\det(A) \neq 0$

$Ax = b$

$a_{11}x_1 + a_{12}x_2 + \dots = b_1$

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}_{n\times n} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

coeff. matr.

A_i : ith column of A is changed with b

$$\boxed{x_i = \frac{\det(A_i)}{\det(A)}}$$

Ex

$$x_1 - 4x_2 + 3x_3 = -2$$
$$3x_1 - x_3 = 5$$
$$2x_1 + x_2 + x_3 = -3$$

$$A = \begin{array}{ccc} + & - & + \\ \end{array}$$
$$A = \begin{bmatrix} 1 & -4 & 3 \\ 3 & 0 & -1 \\ 2 & 1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} -2 \\ 5 \\ -3 \end{bmatrix}$$

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{21}{30} \qquad x_2 = \frac{\det(A_2)}{\det(A)} = \frac{-45}{30} \qquad x_3 = \frac{\det(A_3)}{\det(A)} = \frac{-87}{30}$$

$$\det(A) = 1 \cdot \underbrace{\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}}_{0-(-1)=1} - (-4) \underbrace{\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}}_{3-(-2)=5} + 3 \underbrace{\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}}_{3-0=3}$$
$$= 1 + 20 + 9 = 30$$

unique solution $= \left(\frac{7}{10}, -\frac{3}{2}, -\frac{29}{10}\right)$

$$A_1 = \begin{bmatrix} -2 & -4 & 3 \\ 5 & 0 & -1 \\ -3 & 1 & 1 \end{bmatrix}$$
$\uparrow$ b

$$\det(A_1) = (-2) \underbrace{\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}}_{0-(-1)=1} - (-4) \underbrace{\begin{vmatrix} 5 & -1 \\ -3 & 1 \end{vmatrix}}_{5-3=2} + 3 \underbrace{\begin{vmatrix} 5 & 0 \\ -3 & 1 \end{vmatrix}}_{5-0=5} = -2 + 8 + 15 = 21$$

b $\downarrow$

$$A_2 = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 5 & -1 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\det(A_2) = 1 \cdot \underbrace{\begin{vmatrix} 5 & -1 \\ -3 & 1 \end{vmatrix}}_{5-3=2} - (-2) \underbrace{\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix}}_{3-(-2)=5} + 3 \underbrace{\begin{vmatrix} 3 & 5 \\ 2 & -3 \end{vmatrix}}_{-9-10=-19} = 2 + 10 - 57 = -45$$

$$A_3 = \begin{bmatrix} 1 & -4 & -2 \\ 3 & 0 & 5 \\ 2 & 1 & -3 \end{bmatrix}$$

$$\det(A_3) = 1 \cdot \begin{vmatrix} 0 & 5 \\ 1 & -3 \end{vmatrix} - (-4) \begin{vmatrix} 3 & 5 \\ 2 & -3 \end{vmatrix} + (-2) \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} = -5 - 76 - 6 = -87$$

$$A_3 = \begin{bmatrix} 1 & -4 & -2 \\ 3 & 0 & 5 \\ 2 & 1 & -3 \end{bmatrix} \qquad \det(A_3) = 1 \cdot \underbrace{\begin{vmatrix} 0 & 5 \\ 1 & -3 \end{vmatrix}}_{0-5=-5} - (-4)\underbrace{\begin{vmatrix} 3 & 5 \\ 2 & -3 \end{vmatrix}}_{-9-10=-19} + (-2)\underbrace{\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix}}_{3-0=3} = -5 - 76 - 6 = -87$$